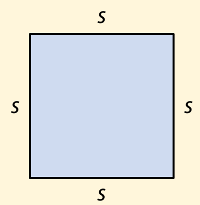
Square Roots and Cube Roots

Use What You Know

In Lesson 1 you learned the properties of integer exponents. Now, take a look at this problem.

The length of each side of a square measures s inches long. The area of the square is 49 in.^{2}. What is the length of one side of the square?



Use the math you know to answer the question.

a. Describe in words how to find the area of the square given that each side is s inches long.

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b. Write a multiplication expression using the variable s to represent the area of the square.

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c. Write an expression using the variable s and an exponent to represent the area of the square. ______________________________

d. Write an equation setting your expression equal to the area of the square given in the problem. ______________________________

e. Consider the factors of 49. Explain what the two sides of the equation have in common when you write each as the product of two factors.

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Find Out More

The number 49 is one of a set of numbers called perfect squares. A perfect square is a number that results from multiplying an integer by itself. The first 15 perfect squares are shown.

$1^2 = 1$	$4^2 = 16$	$7^2 = 49$	$10^2 = 100$	$13^2 = 169$
$2^2 = 4$	$5^2 = 25$	$8^2 = 64$	$11^2 = 121$	$14^2 = 196$
$3^2 = 9$	$6^2 = 36$	$9^2 = 81$	$12^2 = 144$	$15^2 = 225$

Look at the equation you wrote on the previous page, $s^2 = 49$. How do you solve an equation where a variable squared is equal to a perfect square? You have solved equations before by using inverse operations. You solved addition equations by subtracting. You solved division equations by multiplying. What is the inverse operation of squaring a number?

The inverse operation of squaring is finding the **square root**. A square root of a number is any number that you can multiply by itself to get your original number. For example, 3 is a square root of 9, because $3 \cdot 3 = 9$. Another square root of 9 is -3, because $(-3) \cdot (-3) = 9$.

The symbol $\sqrt{}$ means *positive square root*. So, $\sqrt{9} = 3$.

$s^2 = 49$	The inverse of squaring is finding a square root.
$s = \pm\sqrt{49}$	Find the square root of both sides.
$s = \pm 7$	49 is a perfect square.
$s = 7$	The length of one side of the square is 7 inches.

Reflect

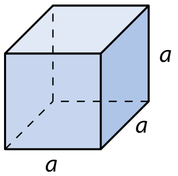
1 What is the difference between dividing 16 by 2 and finding the square roots of 16?

Learn About Finding Cube Roots

Read the problem below. Then explore how to solve equations with cubes and cube roots.

Each edge of a cube measures *a* feet long. The volume of the cube is 125 ft³. What is the measure of each edge of the cube?

Picture It **Draw and label the cube.**



The length, width, and height of the cube each measure *a* feet.

Solve It **You can apply the formula for the volume of a cube.**

The volume of the cube is the product of its length, width, and height.

$a \cdot a \cdot a = V$ length $= a$, width $= a$, and height $= a$

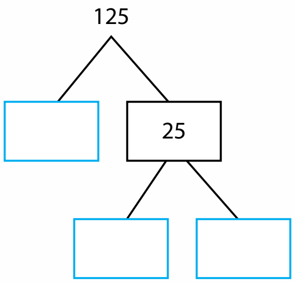
$a^3 = V$

$a^3 = 125$ Substitute the given volume of the cube for V.

You can use this equation to find the value of *a*.

Connect It **Now you will solve the problem from the previous page.**

2 Complete the prime factorization of 125.



3 Write 125 as the product of three factors and as a power of base 5. ______________________

4 What does 125 have in common with a^3 when 125 is written as a power? ________________

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The product of an integer used as a factor three times is a **perfect cube**. Finding the **cube root** is the inverse of cubing a number. The cube root of a number is the number that is used as a factor three times to produce the original number. The symbol $\sqrt[3]{}$ means *find the cube root.*

5 Look at *Solve It* on the previous page. The equation shows a variable cubed equal to a perfect cube.
Use the cube root to complete the solution.

$$a^3 = 125$$

$$\sqrt[3]{a^3} = \sqrt[3]{\boxed{}}$$

$$\sqrt[3]{a^3} = \sqrt[3]{\boxed{}}$$

$$\_\_\_\_ = \_\_\_\_$$

Solution: Each edge of the cube is _____ feet long.

Try It **Use what you just learned to solve these problems. Show your work on a separate sheet of paper.**

6 Solve: $y^3 = 8$ ____

7 Solve: $x^3 = 27$ ____

Learn About Solving Word Problems

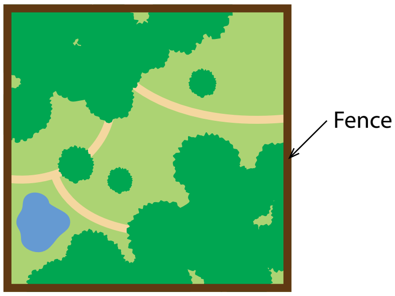
Read the problem below. Then explore how to use square roots and cube roots to solve word problems.

City Park is a square piece of land with an area of 10,000 square yards. What is the length of the fence that encloses the park?

Picture It **You can draw a diagram to help solve the problem.**

The park is a square. The fence runs along the outside edge of the park.

Area $= 10{,}000\text{ yd}^2$



The length of the fence is the perimeter of the square.

Solve It **To find the perimeter of the square park, you need to know the length of one side of the square.**

Let f be the length of one side of the square.

$A = 10{,}000$	Area of the park is $10{,}000\text{ yd}^2$
$f^2 = 10{,}000$	Area equals the length of one side squared.

Connect It **Now you will solve the problem from the previous page.**

8 What number squared equals 10,000? ____________________

9 Look at Solve It on the previous page. Solve the equation for f.

$f^2 = 10{,}000$

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10 What is the length of each side of the park? ____________________

11 Write and solve an equation to find the perimeter of the park. ____________________

12 What is the length of the fence that encloses the park? ____________________

13 The park's rectangular garden area is 450 square yards. Its length is twice its width. Find the dimensions of the garden. Begin with the equation $(2w)(w) = 450$.

Rewrite the equation using exponents. ____________________

Divide both sides by 2. ____________________

Solve and write the garden's dimensions. ____________________

Try It **Use what you just learned about square roots and cube roots to solve these problems. Show your work on a separate sheet of paper.**

14 The volume of a cube is 1,000 cm^3. What is the length of an edge? ____________________

15 A gift box in the shape of a cube has a volume of 216 cm^3. What is the area of the base of the box? ____________________

16 A scientist finds the temperature of a sample at the beginning of an experiment is t°C. After 1 hour, the temperature is t^2°C. If the temperature after 1 hour is 81°C, what are two possible original temperatures? What is the difference between the possible original temperatures? ____________________

Practice Finding Square Roots and Cube Roots

Study the example below. Then solve problems 17–19.

In this problem, you will divide before you find the square root.

Example

The distance in feet that a freely falling dropped object falls in t seconds is given by the equation $\frac{d}{16} = t^2$.

How long does it take a dropped object to fall 64 feet?

Look at how you could solve this problem.

The given equation is: $\frac{d}{16} = t^2$

Substitute 64 for *d*: $\frac{64}{16} = t^2$

Simplify: $4 = t^2$

Because t represents time, t is positive. $\sqrt{4} = \sqrt{t^2}$

Take the positive square root of both sides: $t = 2$

Solution The object takes 2 seconds to fall 64 feet.

Pair/Share
How far does an object fall in 1 second?

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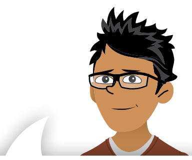
17 The area of the top face of a cube is 9 square meters. What is the volume of the cube?

Show your work.



What information do you need to calculate the volume of a cube?

Pair/Share
The cube has 6 faces. What does the expression $6 \cdot 9$ describe?

Solution ______________________________

18 The length of each edge of a cube is *x* centimeters. If *x* is an integer, why can't the volume of the cube equal 15 cm^3?

Show your work.

Write an equation showing a variable expression for volume is equal to 15.

Solution ______________________________

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Pair/Share

Are all perfect cubes also multiples of 3? Are all multiples of 3 also perfect cubes? Discuss.

19 Yesterday, there were *b* milligrams of bacteria in a lab experiment. Today, there are b^2 milligrams of bacteria. If there are 400 milligrams today, how many milligrams of bacteria were there yesterday?

A 20 milligrams

B 200 milligrams

C 1,600 milligrams

D 160,000 milligrams

Do you square a number or find the square root to solve the problem?

Eva chose **B** as the correct answer. How did she get that answer?

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Pair/Share

Talk about the problem and then write your answer together.

Practice Finding Square Roots and Cube Roots

Solve the problems.

1 Solve $a^3 = 64$.

A $a = 4$

B $a = 8$

C $a = 21$

D $a = 32$

2 Which number is a perfect square?

A 8

B 18

C 200

D 225

3 The fractions below are possible values of x in the given equations. Write the correct fraction inside the box for each equation.

$\frac{9}{8}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{3}$

a. $x^2 = \frac{4}{9}$ $\boxed{}$

b. $x^3 = \frac{27}{64}$ $\boxed{}$

c. $x^2 = \frac{81}{64}$ $\boxed{}$

d. $x^3 = \frac{1}{8}$ $\boxed{}$

4 Use the numbers shown to make the two equations true. Each number can be used only once. Write the number in the appropriate box for each equation.

3	6	100	36	1,000	1,000,000

$\sqrt{\boxed{}} = \boxed{}$ $\quad\quad$ $\sqrt[3]{\boxed{}} = \boxed{}$

5 If x is a positive integer, is $\sqrt{\frac{1}{x^2}}$ greater than, less than, or equal to $\sqrt[3]{\frac{1}{x^3}}$?

Show your work.

Answer ______________________________

6 Describe how you could use inverse operations to solve the equation $\sqrt{x} = 4$.

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✓ Self Check **Go back and see what you can check off on the Self Check on page 1.**