

Understand Rational and Irrational Numbers

Name: _____

Prerequisite: How can you convert fractions to repeating or terminating decimals?



Study the example problem showing how to use division to express fractions as repeating decimals. Then solve problems 1–7.

Example

Erika uses division to write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals. First she estimates that because $\frac{1}{3}$ is between $\frac{1}{4}$ and $\frac{1}{2}$, it will be between 0.25 and 0.5. Likewise, because $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$, it will be between 0.5 and 0.75. Then she divides as shown at the right.

$$\frac{1}{3} = 0.333\dots, \text{ or } 0.\overline{3} \quad \frac{2}{3} = 0.666\dots, \text{ or } 0.\overline{6}$$

0.333	0.666
$3\overline{)1.000}$	$3\overline{)2.000}$
$\underline{-9}$	$\underline{-18}$
10	20
$\underline{-9}$	$\underline{-18}$
10	20
$\underline{-9}$	$\underline{-18}$
1	2

- 1** Erika says that no matter how many decimal places she divides to when she divides 1 by 3, the digit 3 in the quotient will just keep repeating. Is she correct? Explain.

- 2** Is the decimal for $\frac{4}{3}$ a *repeating decimal*? Explain.

- 3** How could Erika have used the decimal that she wrote for $\frac{1}{3}$ to find the decimal for $\frac{2}{3}$?

Vocabulary

repeating decimal a decimal that never ends but instead repeats the same digit or group of digits over and over.

0.333... and 0.1666... are repeating decimals.



Solve.

- 4 Write the decimal for $\frac{1}{8}$. Explain why this decimal is called a *terminating decimal*.

- 5 Tell whether each statement below is true or false. If it is false, write an example that proves the statement is false.

All fractions can be written as repeating decimals.

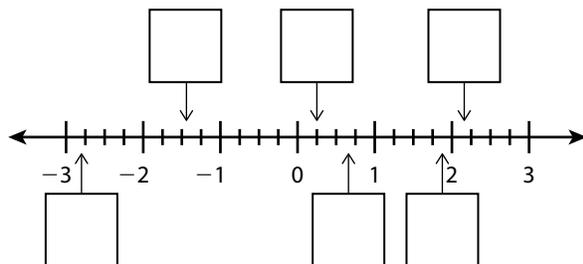
If a fraction can be written as a repeating decimal, only one digit can repeat over and over, without end.

- 6 Raj is playing a game. He needs to find pairs of cards that have the same value. Which two pairs of cards does Raj have that express the same value?



- 7 Write each number in the appropriate box to show its placement along the number line.

-2.8	$\frac{2}{3}$	$2.1\overline{6}$	$1\frac{7}{8}$	0.25	$-\frac{13}{9}$
------	---------------	-------------------	----------------	------	-----------------



Vocabulary

terminating decimal

a decimal that ends, or terminates.

0.5; 4.08; 0.300

Estimate Irrational Numbers

Study the example problem showing how to estimate the value of an irrational number. Then solve problems 1–8.

Example

Estimate the value of $\sqrt{6}$ to the nearest hundredth.

Because $\sqrt{6}$ is between $\sqrt{4}$, which equals 2, and $\sqrt{9}$, which equals 3, $\sqrt{6}$ is between 2 and 3, but it is closer to 2 than to 3.

Find the squares of tenths that are closer to 2 than to 3 in order to find which two tenths $\sqrt{6}$ is between.

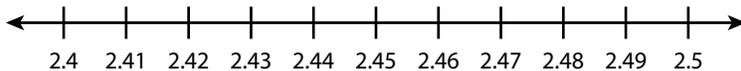
$$2.3^2 = 5.29 \quad 2.4^2 = 5.76 \quad 2.5^2 = 6.25$$

Because 6 is almost exactly halfway between 5.76 and 6.25, $\sqrt{6}$ must be almost exactly halfway between 2.4 and 2.5. Now you can find which two hundredths $\sqrt{6}$ is between.

$$2.44^2 = 5.9536 \text{ and } 2.45^2 = 6.0025$$

$\sqrt{6}$ is between 2.44 and 2.45, but it is closer to 2.45.

- 1** Mark a point at the approximate location of $\sqrt{6}$ to the hundredths place.



- 2** Check your answer by finding $\sqrt{6}$ using a calculator. What is the result on your screen?

- 3** Find $\sqrt{10}$ to the nearest hundredth. Explain how you found your answer.

Vocabulary

irrational number

a number that cannot be expressed as the quotient of two integers. The decimal form never terminates or repeats.

$\sqrt{3}$ is an irrational number.

Solve.

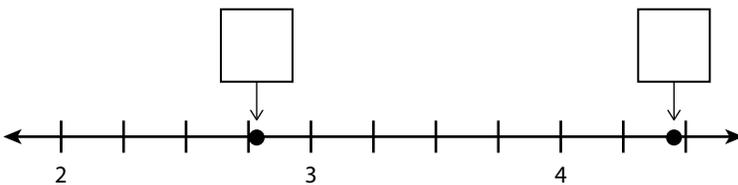
- 4 Explain how a rational number and an irrational number are different.

- 5 Describe how you would compare 3.6 and $\sqrt{12}$.

- 6 Is 1.75 a reasonable estimate of the value of $\sqrt{8}$? Explain your reasoning.

- 7 On a number line, will $\sqrt{20}$ be closer to 4.4 or 4.5? Explain your reasoning.

- 8 Look at the two points on the number line. Each number graphed is the square root of a whole number that is not a perfect square. Write the appropriate square root in each box. Explain how you found your answers.





Vocabulary

rational number a number that can be expressed as the quotient of two integers.

$$2.5 = \frac{25}{10}$$

$$0.8333\dots = \frac{5}{6}$$

Reason and Write

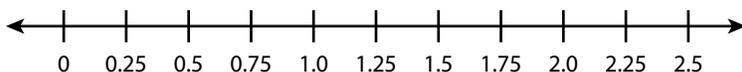
Study the example problem. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

Tell whether the following numbers are rational or irrational, and explain how you know.

$$\frac{1}{9} \quad \sqrt{1.44}$$

Write a decimal approximation for each number, and place it on the number line.



Show your work. Use decimal approximations, a number line, and words to explain your answers.

Possible answer:

$\frac{1}{9}$ is a rational number

because it is the quotient of two integers. When I divide 1 by 9, I get a repeating decimal.

$$\begin{array}{r} 0.111 \\ 9 \overline{) 1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

$\sqrt{1.44}$ is also a rational number. I know that $\sqrt{1.44}$ is between $\sqrt{1}$ and $\sqrt{4}$, or 1 and 2, but it is closer to 1. Then I can find two tenths that $\sqrt{1.44}$ is between: $1.1^2 = 1.21$ and $1.2^2 = 1.44$. Because $1.2^2 = 1.44$, I know that $\sqrt{1.44}$ must be 1.2, which is a rational number.



Where does the example...

- identify whether the numbers are rational or irrational?
- include a decimal approximation?
- include a number line?
- use words to explain?

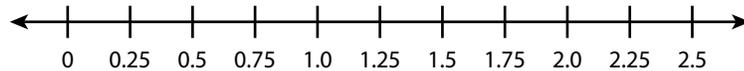


Solve the problem. Use what you learned from the model.

Tell whether the following numbers are rational or irrational, and explain how you know.

$$\sqrt{\frac{2}{8}} \quad \sqrt{2.56}$$

Write a decimal approximation for each number, and place it on the number line.



Show your work. Use decimal approximations, a number line, and words to explain your answers.

Did you ...

- identify whether the numbers are rational or irrational?
- include a decimal approximation?
- include a number line?
- use words to explain?

