

# Multiply and Divide Positive and Negative Integers

## Use What You Know

In Lessons 1–3, you learned that adding integers is a lot like adding and subtracting whole numbers. Take a look at this problem.

What numbers complete Table 1? What numbers complete Table 2?

Table 1	
Multiplication	Product
$3 \cdot 3$	9
$3 \cdot 2$	6
$3 \cdot 1$	3
$3 \cdot 0$	0
$3 \cdot (-1)$	
$3 \cdot (-2)$	
$3 \cdot (-3)$	

Table 2	
Multiplication	Product
$-3 \cdot 3$	
$-3 \cdot 2$	-6
$-3 \cdot 1$	-3
$-3 \cdot 0$	
$-3 \cdot (-1)$	
$-3 \cdot (-2)$	
$-3 \cdot (-3)$	

Use the math you already know to solve the problem.

- Look for a pattern in the products in Table 1. Each product decreases by \_\_\_\_\_.
- I can fill Table 1 by adding \_\_\_\_\_ to each product. Then fill Table 1.
- Why can you use your entry in Table 1 for  $3 \cdot (-3)$  to find the product for  $-3 \cdot 3$  in Table 2?  
\_\_\_\_\_
- Look for a pattern in the products in Table 2. Each product increases by \_\_\_\_\_.
- I can fill Table 2 by adding \_\_\_\_\_ to each product. Then fill Table 2.
- What do you notice about the factors that result in a negative product?  
\_\_\_\_\_
- What do you notice about the product when both factors are negative?  
\_\_\_\_\_

## Find Out More

Multiplying integers is a lot like multiplying whole numbers. You can use what you know about the properties of operations and repeated addition to multiply integers.

Think about these problems:

$-3 \cdot 1 = ?$	Use the identity property.	$-3 \cdot 1 = -3$
$-3 \cdot 0 = ?$	Use the zero product property.	$-3 \cdot 0 = 0$
$-3 \cdot 3 = ?$	Use the commutative property. Multiplying 3 groups of $(-3)$ is the same as adding 3 groups of $-3$ .	$-3 \cdot 3 = 3 \cdot (-3)$ $3 \cdot (-3) = (-3) + (-3) + (-3) = -9$

You can use the distributive property to show why you get a positive product when you multiply two negative numbers. You know  $-3 \cdot 3 = -9$ .

One way you can write  $-3 \cdot 3$  is to think of it as  $-3 \cdot [5 + (-2)]$ .

$$\text{Write 3 as } [5 + (-2)]. \quad -3 \cdot 3 = -3 \cdot [5 + (-2)]$$

$$\text{Use the distributive property.} \quad -9 = (-3)(5) + (-3)(-2).$$

$$\begin{aligned} \text{Think:} \quad & (-3)(5) \text{ is the same as } 5(-3) \text{ or} & -9 = -15 + (-3)(-2) \\ & (-3) + (-3) + (-3) + (-3) + (-3) \\ & = -15 \end{aligned}$$

Since  $-9 = -15 + 6$ , we know that  $(-3)(-2) = 6$ .

So when you multiply two negative numbers, the product is a positive number.

You can use these facts to find products when there are more than two negative numbers.

$$\begin{aligned} (-3) \cdot (-3) \cdot (-3) &= & (-3) \cdot (-3) \cdot (-3) \cdot (-3) &= \\ 9 \cdot (-3) &= -27 & 9 \cdot 9 &= 81 \end{aligned}$$

## Reflect

1 For each statement, tell whether the product is *positive* or *negative*:

When you multiply a positive integer by a negative integer or a negative integer by a positive integer, the product is \_\_\_\_\_.

When you multiply a negative integer by a negative integer, the product is \_\_\_\_\_.

When you multiply an even number of negative integers, the product is \_\_\_\_\_.

When you multiply an odd number of negative integers, the product is \_\_\_\_\_.

**Learn About**  **Multiplying Integers**

Read the problem below. Then explore different ways to understand multiplying integers.

Lisa owes \$2 to each of 6 friends. What integer represents Lisa's debt?

**Model It** You can use groups to help understand the problem.

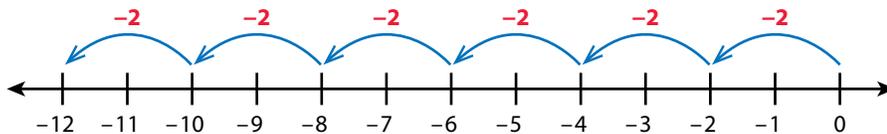
Lisa owes \$2 to each of 6 friends. You can write that as 6 groups of  $(-2)$ .

$$6 \text{ groups of } (-2) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) = -12$$

**Model It** You can also use models to understand the problem.

The following number line shows the amount Lisa owes her friends.

Start at 0 and make 6 jumps of  $-2$ .



**Connect It** Now you will solve the problem from the previous page using equations.

2 What number represents the amount Lisa owes each friend? \_\_\_\_\_

3 Is the amount positive or negative? Why?

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4 How many friends does Lisa owe? \_\_\_\_\_

5 Complete the phrase to show how much Lisa owes.

\_\_\_\_\_ groups of \_\_\_\_\_

6 Rewrite the addition of groups as multiplication of groups.

$$(-2) + (-2) + (-2) + (-2) + (-2) + (-2) = \underline{\hspace{2cm}} \cdot (-2)$$

7 Adding 6 groups of  $(-2)$  is the same as \_\_\_\_\_ 6 and  $(-2)$ .

8 Complete the multiplication to find the integer that represents Lisa's total debt.

$$6 \cdot (-2) = \underline{\hspace{2cm}}$$

9 Explain how you can use the rules of multiplying a positive number by a negative number to check that your answer has the correct sign.

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**Try It** Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

10  $-8 \cdot 12 = \underline{\hspace{2cm}}$

11 A crack in a water tank leaks 7 gallons of water each hour. After 9 hours, what is the change in the volume of water in the tank due to the leak?

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**Learn About**  **Dividing Integers**

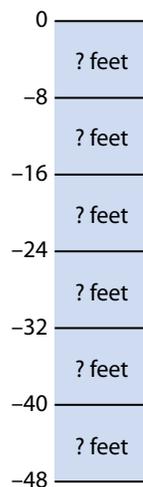
Read the problem below. Then explore different ways to understand dividing integers.

Rosa is at the top of a climbing wall. She descends 48 feet of the wall in 6 equal stages. What number represents the change in Rosa's position at each stage?

**Picture It** You can use a diagram to help you understand the problem.

The following diagram shows the change in Rosa's position at each stage. The top of the wall is the starting or 0 position.

Rosa's Change in Position



**Model It** You can also use division to help you understand the problem.

$$\begin{array}{ccc} \boxed{\text{Total change}} & \div & \boxed{\text{Number of}} \\ \boxed{\text{(in feet)}} & & \boxed{\text{stages}} \\ \hline -48 & \div & 6 \\ \hline & = & \boxed{\text{Change in each stage}} \\ & & \boxed{\text{(in feet)}} \\ & & \hline & & ? \end{array}$$

**Connect It** Now you will solve the problem from the previous page by reasoning about division.

- 12** Rewrite Rosa's division problem from the previous page as a multiplication problem.

$$-48 \div 6 = ? \quad \longrightarrow \quad 6 \cdot ? = \underline{\hspace{2cm}}$$

- 13** Think about what you know about multiplying positive and negative integers. Will the missing number in the multiplication problem be positive or negative? Why?

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- 14** Solve to find the change in Rosa's position at each stage.

$$6 \cdot \underline{\hspace{2cm}} = -48$$

The change in position at each stage is  $\underline{\hspace{2cm}}$  feet.

- 15** Now look at these division problems. Rewrite each as multiplication. Then use multiplication rules to decide whether the missing number is positive or negative.

$$-15 \div 3 = ? \quad \underline{\hspace{2cm}} \cdot ? = \underline{\hspace{2cm}} \quad \text{The missing number is } \underline{\hspace{2cm}}.$$

$$-15 \div (-3) = ? \quad \underline{\hspace{2cm}} \cdot ? = \underline{\hspace{2cm}} \quad \text{The missing number is } \underline{\hspace{2cm}}.$$

$$15 \div (-3) = ? \quad \underline{\hspace{2cm}} \cdot ? = \underline{\hspace{2cm}} \quad \text{The missing number is } \underline{\hspace{2cm}}.$$

- 16** Use the multiplication rules to complete these statements about division with integers.

When you divide a negative integer by a positive integer, the quotient is  $\underline{\hspace{2cm}}$ .

When you divide a negative integer by a negative integer, the quotient is  $\underline{\hspace{2cm}}$ .

When you divide a positive integer by a negative integer, the quotient is  $\underline{\hspace{2cm}}$ .

**Try It** Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

**17**  $54 \div (-9) = \underline{\hspace{2cm}}$

- 18** A glider descends 240 meters in 12 seconds. What is the average change in elevation per second?

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**Practice**  **Multiplying and Dividing Integers**

Study the example below. Then solve problems 19–21.

**Example**

Paul, Hayden, and Alex each have a card with a multiplication sentence written on it. Paul thinks each card has a different solution. Hayden thinks each card has the same solution. Alex thinks only two cards have the same solution. Who is correct?

$$-4 \cdot 3 = ?$$

$$4 \cdot (-3) = ?$$

$$-4 \cdot (-3) = ?$$



The student used multiplication rules to check the sign for each solution.

Look at how you could show your work using multiplication rules.

Equation	Rule	Solution
$-4 \cdot 3 = ?$	negative $\cdot$ positive = negative	-12
$4 \cdot (-3) = ?$	positive $\cdot$ negative = negative	-12
$-4 \cdot (-3) = ?$	negative $\cdot$ negative = positive	+12

**Solution** Alex is correct. Only two cards have the same solution, -12.

**Pair/Share**

How could you use addition of groups to show that  $4 \cdot (-3) = -12$ ?

- 19** A movie rental company deducts \$7 from Serena's account every month for a year. After 1 year, how much do movie rental fees change Serena's account?

**Show your work.**



Will your answer be positive or negative?

**Pair/Share**

How can you check that your answer is reasonable?

**Solution** \_\_\_\_\_

- 20 The table shows the low temperatures in Alto for 5 days. What is the average low temperature?

Daily Low Temperature in Alto	
Day	Low Temperature
1	$-4^{\circ}\text{F}$
2	$-7^{\circ}\text{F}$
3	$-13^{\circ}\text{F}$
4	$-5^{\circ}\text{F}$
5	$-5^{\circ}\text{F}$

Show your work.

**Solution** \_\_\_\_\_

- 21 Which equation has a negative solution?

- A  $-6 \div (-2) = \square$
- B  $-6 + (-2) = \square$
- C  $-2 \cdot (-6) = \square$
- D  $-2 - (-6) = \square$

Seth chose **C** as the correct answer. How did he get that answer?

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To find the average of a set of numbers, do you multiply or divide?

**Pair/Share**

How did you and your partner decide what step to do first?



How can you use the multiplication rules to help you solve this problem?

**Pair/Share**

How did you and your partner choose your answer?

**Practice**  **Multiplying and Dividing Integers****Solve the problems.****1** Which multiplication equation is **false**?

- A  $(-a) \cdot (-1) = (-a)$
- B  $(-a) \cdot 0 = 0$
- C  $(-a) \cdot 1 = (-a)$
- D  $(-a) \cdot b = b \cdot (-a)$

**2** The lowest elevation in Long Beach, California, is 7 feet below sea level. The elevation of Death Valley is about 40 times lower than the elevation of Long Beach. What is the approximate elevation of Death Valley?

- A -280 feet
- B -47 feet
- C -33 feet
- D -6 feet

**3** Draw a line from each expression to the word problem it could represent.

$-12 \times 4$	A football team loses a total of 12 yards in 4 plays. On average, how many yards did the team move per play?
$12 \times 4$	The temperature of the ocean water dropped 4 degrees every hour. After 12 hours, what was the change in the water temperature?
$12 \times (-4)$	A garden consists of 12 rose bushes in each of 4 rows. How many rose bushes does the garden have altogether?
$-3 \times (-4)$	Every second, a snail crawls 12 millimeters down a hole. What is the snail's change in position after 4 seconds?
$-12 \div 4$	During a drought, a pond's fish population decreased by 3 for every meter drop in the pond's water level. At that rate, how many fish died once the water level dropped 4 meters?

- 4 Write each expression under the category that correctly describes the number to which the expression simplifies.

$- (8 \div 2)$   
 $-5 \times (-4 + 3)$   
 $6 \div (-3)$   
 $-10 \div (-10)$   
 $-25 \times (-4)$   
 $-21 \div 7$

**A negative number**

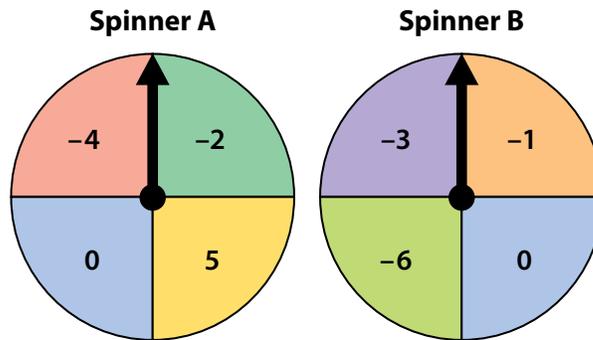
**A positive number**

- 5 Marc's elevation decreases by 350 meters as he rides his bike down to the bottom of a hill. He did the ride in 5 equal stages. How much did Marc's elevation change during each stage?

**Show your work.**

**Answer** \_\_\_\_\_

- 6 Hugh is playing a game. He spins each spinner and then multiplies the numbers to find the product.



**Part A** Give an example of two numbers Hugh could spin to get a positive product.

**Answer** \_\_\_\_\_

**Part B** Give an example of two numbers Hugh could spin to get a negative product.

**Answer** \_\_\_\_\_

**Self Check** Go back and see what you can check off on the Self Check on page 1.